Analytic Tableaus for Natural Logic

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The Fregean revolution in logic, which turned the subject into the thriving theory it is today, also resulted in a move away from language. Frege thought ordinary language too vague and imprecise to be amenable to scientific treatment. This was a break with views held previously: in traditional logic Latin and other languages, only slightly regimented, had always been the main vehicle of representation.

A new convergence between logic and language had to await the work of Richard Montague, who showed how essentially Fregean methods can be used to treat fragments of English in a precise way. But the paradigmatic PTQ paper gives no calculus that works directly on the language itself and requires translation of English into a logical language. Proofs are available only indirectly, via this translation. This contrasts with the situation in logic, where it is customary to provide logics with their own calculus. If there truly is no important difference between natural languages and logical languages, as Montague famously or infamously wrote, it should be possible to find a calculus for the entailment relation in ordinary language that uses only linguistic forms and employs only rules that are linguistically relevant. Finding such a logic also seems highly urgent from a cognitive point of view, for, while people obviously engage in reasoning, some of it slow and concious, some fast and subconcious, it is unlikely that the forms we encounter in standard logic have any cognitive or linguistic significance.

Calculi for the entailment relation in natural language that are based on linguistic representations are studied in *Natural Logic*, the continuation of traditional logic with modern means[1] and this paper will fall within this line of research. We will subscribe to one of the leading ideas often accepted in the natural logic tradition: that in a linguistic context some forms of inference are more natural than others. In particular it seems that reasoning on the basis of *monotonicity* and related algebraic concepts is deeply entrenched within the logical-linguistic system. Such forms of reasoning tend to come easy to language users and often correlate strongly with judgments on syntactic well-formedness. The best known example of such a correlation is Ladusaw's (1979) observation that negative polarity items need licencing by a downward entailing context, but since Ladusaw's early work a host of similar observations have strengthened the general point.

The aim of this paper will be to explore a tableau (or 'truth tree') system for natural language that works directly with lambda terms that do not contain any logical constants. Such lambda terms, I will argue, can very well stand proxy for linguistic representations, as they are close to the linguist's Logical Forms, essentially trees with gaps and binding. I will argue that the methods employed in this paper scale up to much larger parts of the language than those that can reasonably be treated here.

An example of a tableau deduction according to our system, showing that *most students who Mary kissed moved* follows from *each person who Mary touched ran*, is given in Table 1. The tableau entries are each of the form $T\vec{c} : A$ or $F\vec{c} : A$, where A is a *Lambda Logical Form* and $\vec{c}$ is a sequence of constants such that $A\vec{c}$ is

[1] The research line this paper is based upon starts with Van Eijck (1985), van Benthem (1986, 1991), and Sánchez (1991).

$T i$: **each**(**who**(λx.**Mary**(**touched** x))**person**)**ran**
$F i$: **most**(**who**(λx.**Mary**(**kissed** x))**student**)**moved**

$T cj$: **ran**
$F cj$: **moved**
$\times$

TPi : **each**(**who**(λx.**Mary**(**touched** x))**person**)
FPi : **most**(**who**(λx.**Mary**(**kissed** x))**student**)

Tcj : **who**(λx.**Mary**(**kissed** x))**student**
Fcj : **who**(λx.**Mary**(**touched** x))**person**
Tcj : λx.**Mary**(**kissed** x)
Tcj : **student**
Tj : **Mary**(**kissed** c)

Fcj : λx.**Mary**(**touched** x)
Fj : **Mary**(**touched** c)

Tbk : **kissed** c
Fbk : **touched** c
$Tcbk$: **kissed**
$Fcbk$: **touched**
$\times$

$TP'j$: **Mary**
$FP'j$: **Mary**
$\times$

Fcj : **person**
$\times$

$TP'Pi$: **each**
$FP'Pi$: **most**
$\times$

Table 1: Example of a natural logic tableau.

of type t. $T\vec{c} : A$ ($F\vec{c} : A$) means that $A\vec{c}$ is true (false). In particular, the tableau investigates the possibility that **each**(**who**(λx.**Mary**(**touched** x))**person**)**ran** is true in some world i while **most**(**who**(λx.**Mary**(**kissed** x))**student**)**moved** is false in i. The reasoning—which refutes that possibility—makes use of monotonicity rules such as the following.

(1)
$$\begin{array}{c} T\vec{a} : GA \\ F\vec{a} : HB \\ \hline \begin{array}{cc} T\vec{c} : A & Tb\vec{a} : G \\ F\vec{c} : B & Fb\vec{a} : H \end{array} \end{array}$$
where $\vec{c}$ and b are fresh, provided G or H is mon$\uparrow$

This rule is sound if G or H is upward monotone. It is used twice in the tableau, once using the fact that **each N** is mon$\uparrow$ for any **N**, and once using the upward monotonicity of **Mary**. A dual rule, for downward monotone operators, can also easily be formulated, and is used in the tableau on the basis of the downward monotonicity of **each**. The tableau also uses some propositional rules (with **who** acting as a conjunction). Closure of branches is obtained by outright contradictions here, but also with the help of basic hyponomy relations, such as **kissed** $\leq$ **touched**, or **each** $\leq$ **most**.

The paper will investigate a host of other rules, many connected with properties that have arisen in the semantic literature (such as anti-additivity). It will also be argued that the formal system presented here squares well with the psychological view that reasoning is crucially dependent on systematic search for verifying situations (see e.g. Johnson-Laird, 1983, 2006, and the many references therein).

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